

**TL; DR:** Design a simple yet effective subgraph sampling method for GNNs with historical embeddings [1][2][3]

- Based on theoretical analysis of the approximation error caused by using historical embeddings for out-of-batch neighbors
- Better performance compared to the default sampling method (as in Cluster-GCN[4])
- Do not bring additional computation overhead due to efficient staleness score calculation, improved re-sampling strategy, and faster training converge

## Background - Training GNNs on large-scale graphs

- Mini-batch training is necessary
- Sampling-based methods are commonly used (our focus in this paper), basically we sample a subgraph as a mini-batch
  - There may (e.g. node/layer-wise sampling) or may not (e.g. cluster-based) be overlapping nodes between different subgraphs
- Another line of work is treating each node as an training example and do mini-batch training with fixed batch size, but each node is attached with fixed-dimensional information [5] (e.g. top-k ppr neighbors, top-k hop aggregated embeddings)

## Background - GNNs with historical embeddings

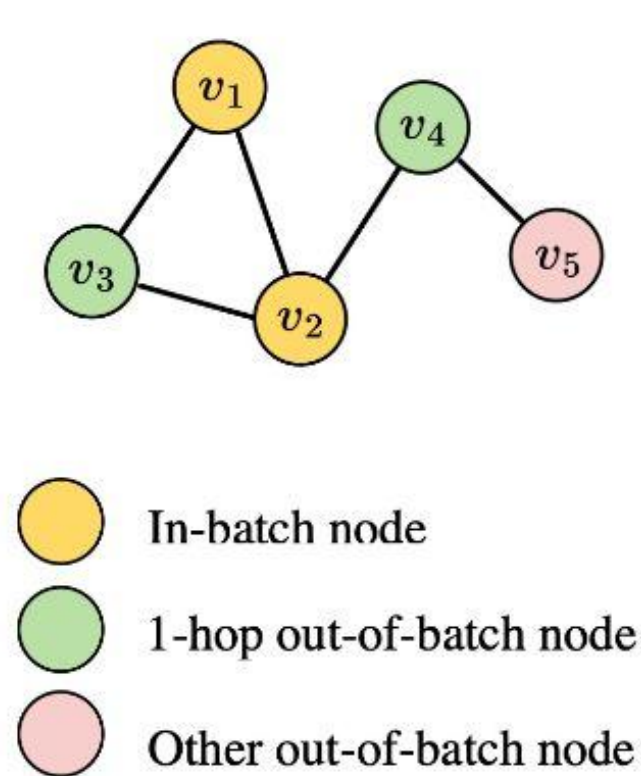
- Use historical embeddings for the unsampled neighbors

$$\begin{aligned} h_u^{l+1} &= f_{\theta}^{l+1}(h_u^l, \{h_v^l\}_{v \in \mathcal{N}(u)}) \\ &= f_{\theta}^{l+1}(h_u^l, \{h_v^l\}_{v \in \mathcal{N}(u) \cap \mathcal{B}} \cup \{\bar{h}_v^l\}_{v \in \mathcal{N}(u) \setminus \mathcal{B}}) \\ &\approx f_{\theta}^{l+1}(h_u^l, \{h_v^l\}_{v \in \mathcal{N}(u) \cap \mathcal{B}} \cup \{\bar{h}_v^l\}_{v \in \mathcal{N}(u) \setminus \mathcal{B}}) \end{aligned}$$

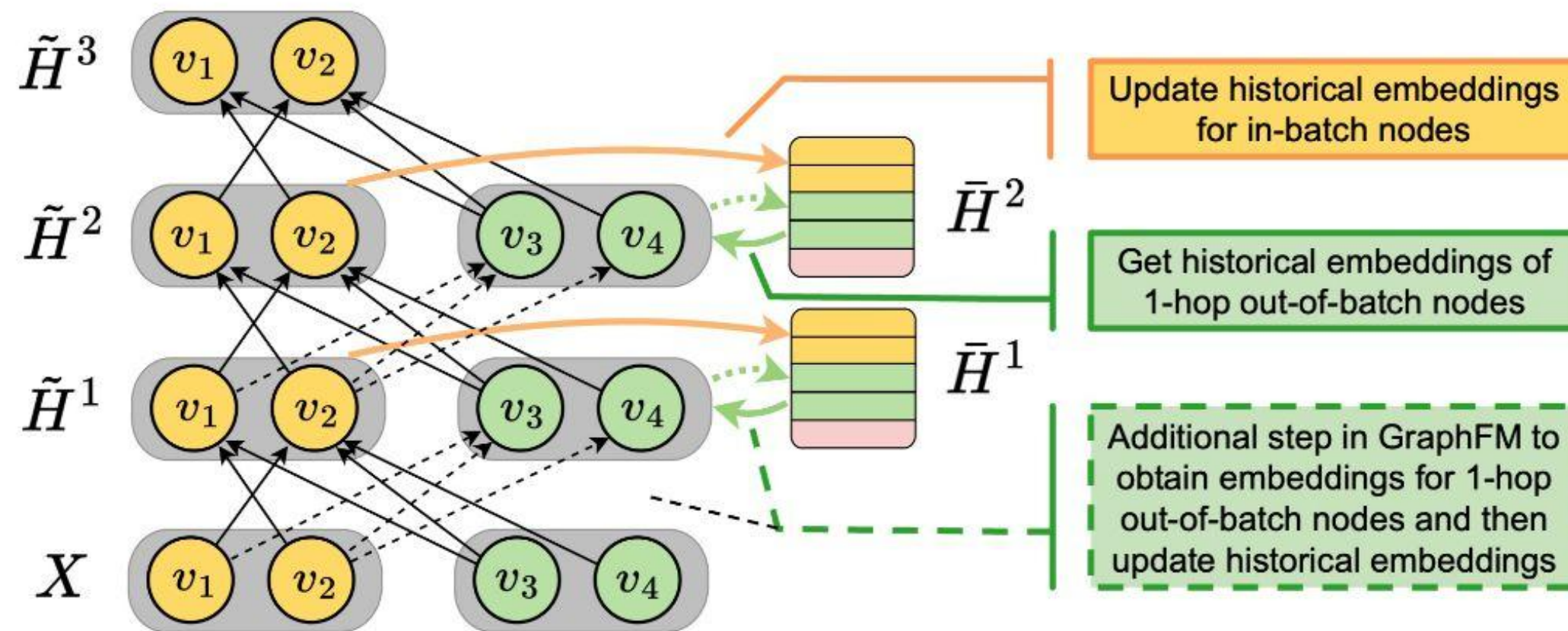
$v \in \mathcal{N}(u) \setminus \mathcal{B}$  : unsampled neighbors

$v \in \mathcal{N}(u) \cap \mathcal{B}$  : sampled neighbors

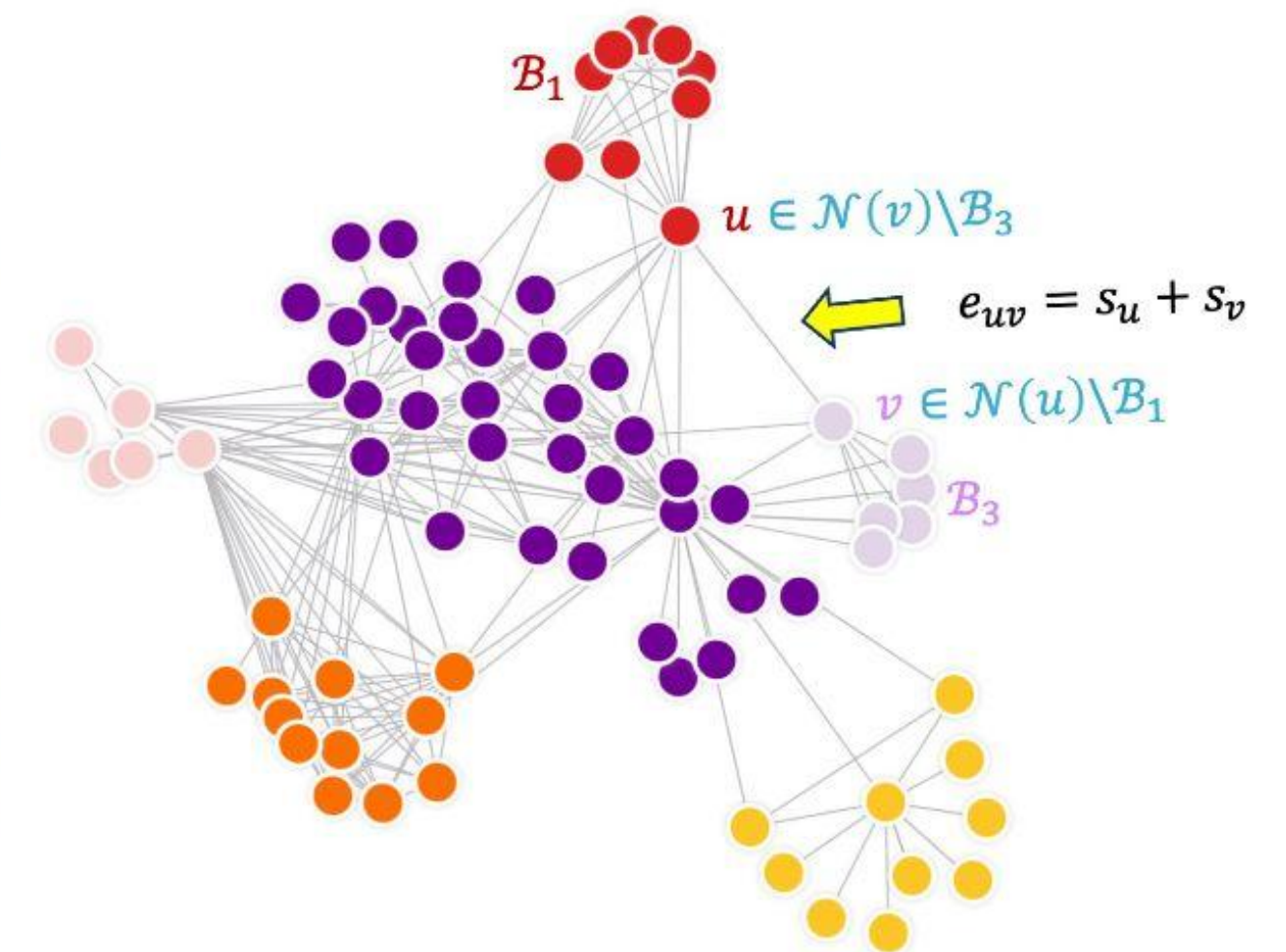
$\bar{h}_v^l$  : historical embeddings



Example graph



Historical embedding-based methods



Staleness-based subgraph sampling

## Method - Staleness-based subgraph sampling

- Minimize the approximation error from using historical embeddings
$$\|h_u^L - \tilde{h}_u^L\|_2^2$$
- Equivalent to minimize the weighted sum of the staleness scores for all out-of-batch neighbors
$$\arg \min_{\mathcal{B}} \sum_{u \in \mathcal{B}} \sum_{v \in \mathcal{N}(u) \setminus \mathcal{B}} \sum_{\ell} C_v^{\ell} s_v^{\ell}$$

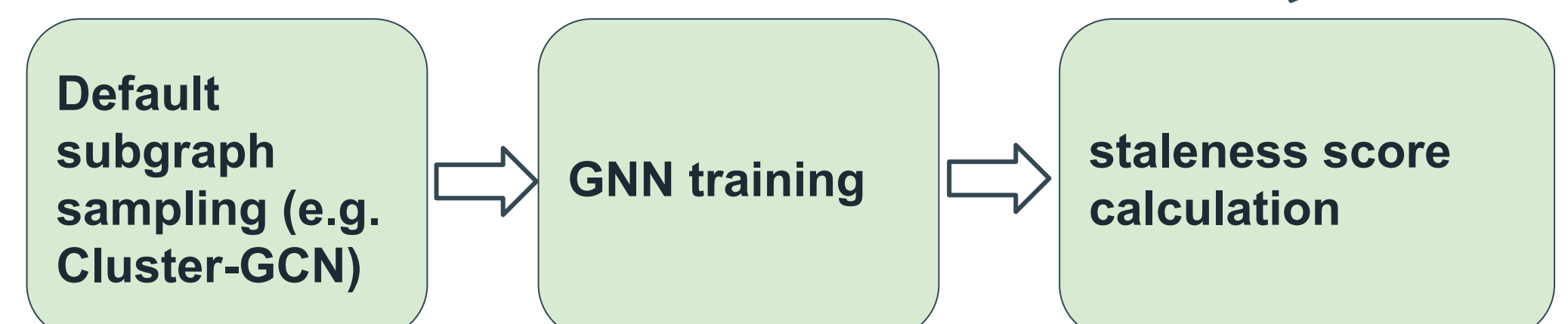
where the staleness score  $s_v^{\ell} = \|h_v^{\ell} - \bar{h}_v^{\ell}\|$

- Considering all M mini-batches, the overall minimization objective is

$$\begin{aligned} \arg \min_{\{\mathcal{B}_1, \dots, \mathcal{B}_M\}} & \sum_{\mathcal{B}_i \in \{\mathcal{B}_1, \dots, \mathcal{B}_M\}} \sum_{u \in \mathcal{B}_i} \sum_{v \in \mathcal{N}(u) \setminus \mathcal{B}_i} \sum_{\ell} C_v^{\ell} s_v^{\ell} \\ \text{subject to } & \mathcal{V} = \mathcal{B}_1 \cup \mathcal{B}_2 \cup \dots \cup \mathcal{B}_M \\ & \mathcal{B}_i \cap \mathcal{B}_j = \emptyset \text{ for all } i \neq j, 1 \leq i, j \leq M \end{aligned}$$

- Equivalent to graph partitioning objective [6] (to the right)

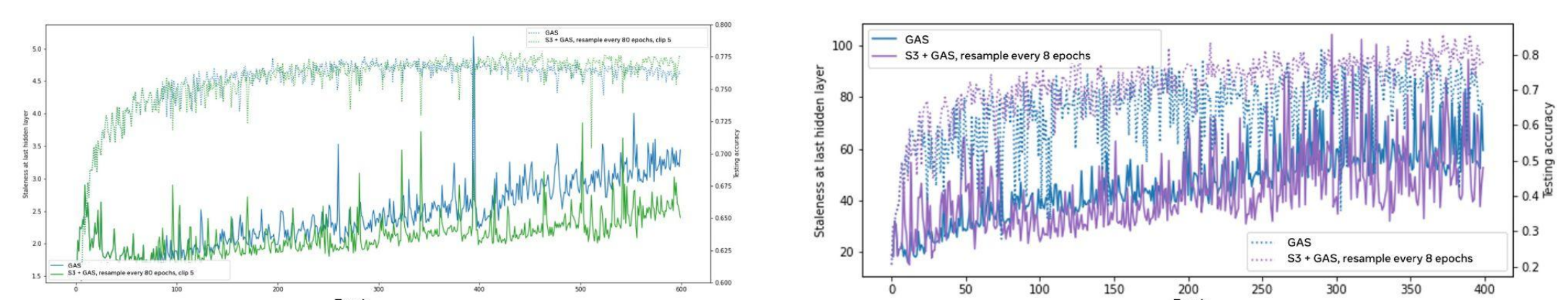
Re-sampling scheduler discussed in the experimental results part



$$\begin{aligned} \arg \min_{\{\mathcal{B}_1, \dots, \mathcal{B}_M\}} & \sum_{u \in \mathcal{B}_i, v \in \mathcal{B}_j, i \neq j, (u, v) \in \mathcal{E}} \sum_{\ell} C_u^{\ell} s_u^{\ell} + C_v^{\ell} s_v^{\ell} \\ \text{subject to } & \mathcal{V} = \mathcal{B}_1 \cup \mathcal{B}_2 \cup \dots \cup \mathcal{B}_M \\ & \mathcal{B}_i \cap \mathcal{B}_j = \emptyset \text{ for all } i \neq j, 1 \leq i, j \leq M \end{aligned}$$

## Experimental results

# Nodes # Edges Method	89K 450K Flickr	230K 11.6M Reddit	717K 7.9M Yelp	169K 1.2M ogbn-arxiv	2.4M 61.9M ogbn-products
VR-GCN	0.482 ± 0.003	0.964 ± 0.001	0.640 ± 0.002	—	—
FastGCN	0.504 ± 0.001	0.924 ± 0.001	0.265 ± 0.053	—	—
GraphSAINT	0.511 ± 0.001	0.966 ± 0.001	0.653 ± 0.003	—	0.791 ± 0.002
Cluster-GCN	0.481 ± 0.005	0.954 ± 0.001	0.609 ± 0.005	—	0.790 ± 0.003
SIGN	0.514 ± 0.001	0.968 ± 0.000	0.631 ± 0.003	0.720 ± 0.001	0.776 ± 0.001
GraphSAGE	0.501 ± 0.013	0.953 ± 0.001	0.634 ± 0.006	0.715 ± 0.003	0.783 ± 0.002
GCN	0.534 ± 0.001	0.954 ± 0.000	—	0.715 ± 0.002	0.767 ± 0.002
S3 + GAS	<b>0.545 ± 0.001</b>	<b>0.955 ± 0.000</b>	—	<b>0.724 ± 0.002</b>	<b>0.771 ± 0.002</b>
GCNII	0.554 ± 0.003	0.967 ± 0.000	0.639 ± 0.003	0.725 ± 0.003	0.770 ± 0.002
S3 + GAS	<b>0.567 ± 0.002</b>	<b>0.969 ± 0.001</b>	<b>0.652 ± 0.003</b>	<b>0.735 ± 0.002</b>	<b>0.778 ± 0.002</b>
GCN	0.535 ± 0.002	0.953 ± 0.000	—	0.715 ± 0.003	0.767 ± 0.001
FM	0.549 ± 0.001	0.952 ± 0.000	—	<b>0.722 ± 0.002</b>	<b>0.770 ± 0.002</b>
S3 + FM	<b>0.549 ± 0.001</b>	0.952 ± 0.000	—	<b>0.722 ± 0.002</b>	<b>0.770 ± 0.002</b>
GCNII	0.547 ± 0.003	0.965 ± 0.001	0.641 ± 0.003	0.725 ± 0.003	0.771 ± 0.002
S3 + FM	<b>0.566 ± 0.003</b>	<b>0.969 ± 0.000</b>	<b>0.652 ± 0.002</b>	<b>0.733 ± 0.003</b>	<b>0.776 ± 0.001</b>
GCN	0.538 ± 0.001	0.954 ± 0.000	—	0.714 ± 0.002	0.765 ± 0.002
LMC	<b>0.541 ± 0.001</b>	<b>0.955 ± 0.000</b>	—	<b>0.721 ± 0.002</b>	<b>0.770 ± 0.002</b>
S3 + LMC	<b>0.541 ± 0.001</b>	<b>0.955 ± 0.000</b>	—	<b>0.721 ± 0.002</b>	<b>0.770 ± 0.002</b>
GCNII	0.554 ± 0.005	0.969 ± 0.000	0.647 ± 0.003	0.728 ± 0.002	0.769 ± 0.002
LMC	<b>0.562 ± 0.002</b>	0.969 ± 0.000	<b>0.650 ± 0.003</b>	<b>0.731 ± 0.001</b>	<b>0.773 ± 0.002</b>
S3 + LMC	<b>0.562 ± 0.002</b>	0.969 ± 0.000	<b>0.650 ± 0.003</b>	<b>0.731 ± 0.001</b>	<b>0.773 ± 0.002</b>



Comparison between GAS (blue) and S3 + GAS (green and purple) on ogbn-products and Reddit datasets in terms of staleness scores (solid line) and testing accuracy (dashed line).

		Flickr & GCNII	ogbn-arxiv & GCNII
Epochs	LMC	356	197.4
	S3 + LMC	<b>211.4</b>	<b>180</b>
Runtime (s)	LMC	475	178
	S3 + LMC	<b>304</b>	<b>175</b>

Running time (s)	ogbn-arxiv	ogbn-products
Training per epoch	1	37
Staleness score calculation	0	3
Re-sampling from scratch	3	120
Re-sampling with fast refinement	2	48

S3 + GAS	No resampling	Resampling once	Resample every a fixed number of epochs				
			80	40	20	8	1
Flickr	0.5667	0.5683	<b>0.5729</b>	0.5711	0.5692	0.5715	0.5703
ogbn-arxiv	0.7250	0.7278	0.7291	0.7300	<b>0.7303</b>	0.7294	0.7292
ogbn-products	0.7991	0.8001	0.8030	<b>0.8069</b>	0.8035	—	—

[1] M. Fey et al. "GNNAutoScale: Scalable and expressive graph neural networks via historical embeddings." ICML 2021

[2] H. Yu et al. "GraphFM: Improving large-scale gnn training via feature momentum." ICML 2022

[3] Z. Shi et al. "LMC: Fast training of GNNs via subgraph sampling with provable convergence." ICLR 2023

[4] W.-L. Chiang et al. "Cluster-GCN: An efficient algorithm for training deep and large graph convolutional networks." KDD 2019

[5] Fu, Dongqi, et al. "VCR-Graphormer: A mini-batch graph transformer via virtual connections." ICLR 2024

[6] G. Karypis et al. "A fast and high quality multilevel scheme for partitioning irregular graphs." SIAM Journal on scientific Computing 1998